

Special Relativity in Reduced Power Algebras

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Abstract

Recently, [10,11], the Heisenberg Uncertainty relation and the No-Cloning property in Quantum Mechanics and Quantum Computation, respectively, have been extended to versions of Quantum Mechanics and Quantum Computation which are re-formulated using scalars in *reduced power algebras*, [2-9], instead of the usual real or complex scalars. Here, the Lorentz coordinate transformations, fundamental in Special Relativity, are extended to versions of Special Relativity that are similarly re-formulated in terms of scalars in reduced power algebras, instead of the usual real or complex scalars. The interest in such re-formulations of basic theories of Physics are due to a number of important reasons, [2-11]. Suffice to mention two of them : the difficult problem of so called "infinities in Physics" falls easily aside due to the presence of infinitesimal and infinitely large scalars in such reduced power algebras, and the issue of fundamental constants in physics, like Planck's h , or the velocity of light c , comes under a new focus which offers rather surprising alternatives.

1. A Well Known Usual Deduction of the Lorentz Coordinate Transformations

The following elementary way to obtain the Lorentz coordinate transformations is well known, [1]. Given two coordinate systems S and S' with respective coordinates (x, t) and (x', t') in which the space x -axis and x' -axis are along the same line. We suppose that at time $t = t' = 0$ the origins O and O' of the two coordinate systems coincide, thus $x = x' = 0$. Let now S' move along the x -axis in the positive direction with the constant velocity v , and let two observers be respectively at O and O' .

In that setup, at the initial moment $t = t' = 0$ and when O and O' coincide, a light signal is emitted from O . Its propagation within S is then given by

$$(1.1) \quad x^2 = c^2 t^2$$

where $c > 0$ is the velocity of light.

Now, in view of the Principle of Constancy of the Velocity of Light, in the coordinate system S' the propagation of that light signal is according to

$$(1.2) \quad x'^2 = c^2 t'^2$$

Consequently, one must have

$$(1.3) \quad x^2 - x'^2 = c^2(t^2 - t'^2)$$

However, at least for small values of v , when compared with c , we must have

$$(1.4) \quad x' = k(c, v)(x - vt)$$

for some positive $k(c, v) \in \mathbb{R}$ that does not depend on x, t, x', t' , and which in addition is such that

$$(1.5) \quad \lim_{v \rightarrow 0} k(c, v) = 1$$

since (1.4) and (1.5) are implied by the respective non-relativistic

Galilean coordinate transformation.

Now in view of the Principle of Relativity of Motion, we can suppose that S' is fixed, and S is moving along the x' -axis and in the negative direction, with velocity $-v$. In that case, similar with (1.4), we obtain

$$(1.6) \quad x = k(c, v)(x' + vt')$$

By squaring (1.4) and (1.6), we obtain

$$(1.7) \quad x'^2 + k(c, v)^2 x^2 - 2k(c, v)xx' = k(c, v)^2 v^2 t^2$$

$$(1.8) \quad x^2 + k(c, v)^2 x'^2 - 2k(c, v)xx' = k(c, v)^2 v^2 t'^2$$

thus by subtracting the (1.7) from (1.8), it follows that

$$(1.9) \quad (x^2 - x'^2)(k(c, v)^2 - 1) = k(c, v)^2 v^2 (t^2 - t'^2)$$

and then in view of (1.3), we obtain

$$(1.10) \quad c^2(k(c, v)^2 - 1) = k(c, v)^2 v^2$$

or

$$(1.11) \quad (c^2 - v^2)k(c, v)^2 = c^2$$

In this way

$$(1.12) \quad k(c, v) = c/(c^2 - v^2)^{1/2} = 1/(1 - v^2/c^2)^{1/2}$$

which obviously satisfies (1.5).

The *space coordinate* Lorentz transformation results now from (1.4) and (1.12), namely

$$(1.13) \quad x' = (x - vt)/(1 - v^2/c^2)^{1/2}$$

In order to obtain the *time coordinate* Lorentz transformation, it will be convenient to proceed in full algebraic detail, and with a special

attention to the operations of *division* and *square root* involved. For that purpose, we replace x' in (1.6) with its value from (1.4). The result is

$$(1.14) \quad \begin{aligned} x &= k(c, v)(k(c, v)(x - vt) + vt') = \\ &= k(c, v)^2 x - k(c, v)^2 vt + k(c, v)vt' \end{aligned}$$

or

$$(1.15) \quad k(c, v)vt' = k(c, v)^2 vt - (k(c, v)^2 - 1)x$$

Thus dividing by $k(c, v)v$, one has

$$(1.16) \quad t' = k(c, v)t - (k(c, v)^2 - 1)/(k(c, v)v)x$$

Dividing in (1.11) by $c^2 - v^2$, results that

$$(1.17) \quad k(c, v)^2 = c^2/(c^2 - v^2)$$

and then

$$(1.18) \quad k(c, v)^2 - 1 = v^2/(c^2 - v^2)$$

Now (1.16), (1.12) yield the desired *time coordinate* Lorentz transformation

$$(1.19) \quad t' = (t - vx/c^2)/(1 - v^2/c^2)^{1/2}$$

2. Extending the Lorentz Coordinate Transformations to Reduced Power Algebras

Let us consider instead of the field \mathbb{R} of usual real numbers an arbitrary reduced power algebra $\mathbb{A}_{\mathcal{F}}$, see (A.1.4) in the Appendix. In other words, we shall model both space and time with such algebras $\mathbb{A}_{\mathcal{F}}$, instead of modelling them with the field \mathbb{R} of usual real numbers. Here it is important to note that, in general, such algebras $\mathbb{A}_{\mathcal{F}}$ need *not* be linearly or totally ordered, see (A.4.1) - (A.4.4) in the Appendix.

Furthermore, when they are not linearly or totally ordered, that is, when the respective filters \mathcal{F} are not ultrafilters, then the corresponding algebras $\mathbb{A}_{\mathcal{F}}$ need *not* be one dimensional vector spaces, as is of course the case of \mathbb{R} .

It follows that the extension of the Lorentz coordinate transformations to reduced power algebras opens up a rather wide realm, one in which time, as much as each individual coordinate, may be *multi-dimensional*, and in fact, even *infinite dimensional*.

Speculations regarding the possible meaning of such considerable extensions can, therefore, be diverse and rather numerous. One of them, coming from the multi-dimensionality of time, may be that it could possibly model *parallel universes* ...

And now, let us return to the aimed extension the Lorentz coordinate transformations to arbitrary reduced power algebras.

In this regard, it is sufficient to note that all the algebraic operations in section 1 above, operations leading to the usual Lorentz coordinate transformations in (1.13), (1.19), can automatically be replicated in all the reduced power algebras $\mathbb{A}_{\mathcal{F}}$, except when divisions and square roots are involved. Indeed, when divisions are involved in these algebras one has to consider the presence in them of *zero divisors* and *non-invertible* elements, see section A.2. in the Appendix. As for square roots, one has to proceed according to section A.5. in the Appendix.

3. Comments

3.1. Why Hold to the Archimedean Axiom ?

It is seldom realized, especially among physicists, that ever since ancient Egypt and the axiomatization of Geometry by Euclid, we keep holding to the Archimedean Axiom. This axiom, in simplest terms, such as of a partially ordered group G , for instance, means the following property

$$(3.1.1) \quad \exists u \in G, u \geq 0 : \forall x \in G : \exists n \in \mathbb{N} : x \leq nu$$

or in other words, there exists a "path length" u , so that every element x in the group can be "overtaken" by a finite number n of "steps" of "length" u . Clearly, if G is the set \mathbb{R} of usual real numbers considered with the usual addition, then one can take as u any positive number. As is known, Geometry in ancient Egypt was important in connection with the yearly flood of the Nile and the subsequent need to redraw the boundaries of agricultural land. And for such a purpose, the Archimedean Axiom is obviously useful.

The question, however, is :

Why hold to that axiom when dealing with such modern and highly non-intuitive theories of Physics, as Special and General Relativity, or Quantum Mechanics and Quantum Field Theory ?

Is there any physical type reason in such modern theories for holding to the Archimedean Axiom ?

Indeed, one of the inevitable consequences of the Archimedean Axiom is that "infinity" is not a usual scalar, be it real or complex. Thus all usual algebraic and other operations do rather as a rule break down when reaching "infinity". And this elementary and inevitable fact leads to the long festering problem of the so called "infinities in Physics", a problem which is attempted to be dealt with by various "re-normalization" methods, or by what is an exceedingly complex and so far not yet successful venture, namely, String Theory.

On the other hand, the moment one simply frees oneself from the Archimedean Axiom, and starts to deal with scalars such as those given by various reduced power algebras, the mentioned troubles with "infinity" disappear. Indeed, since the Archimedean Axiom is no longer present in such algebras, these algebras have a rich structure of "infinitesimals" and "infinitely large" scalars, all of which are subjected to the usual algebraic and other operations, just as if they were usual real or complex numbers.

3.2. Two Alternatives When Freed From the Archimedean Axiom

The above way the Lorentz Coordinate Transformations have been extended to space-times built upon scalars given by reduced power algebras may at first seem to be both trivial and without interest. And the same appearance may arise with the extension to such space-times of the Heisenberg Uncertainty and No-Cloning, in [10], respectively, [11].

Here however, one should note the following.

First, even the multiplication in such reduced power algebras is no longer trivial. Indeed, such algebras can have zero divisors, see section A.2. in the Appendix. Consequently, it may easily happen that, although $c, v, x, t, k(c, v) \neq 0$, we will nevertheless have the products in which such quantities appear, and the respective products vanish, contrary to what happens in the usual case when scalars given by real numbers are employed. And clearly, such a vanishing of certain products may invalidate subsequent formulas, or at best, give them a different meaning from the usual one.

Also, mathematical expressions in various theories of Physics can contain operations other than mere multiplication, and such operations can have new properties and meanings, when performed in reduced power algebras.

Therefore, here, we may obviously face two rather different alternatives, namely

- the new properties and meanings in reduced power algebras do not correspond to any possible physical meaning,

or on the contrary

- such new properties and meanings which appear in reduced power algebras may possibly correspond to not yet explored physical realities.

We shall in the sequel mention several possible such new physical interpretations, if not in fact, possible realities.

3.3. Increased and Decreased Precision in Measurements

As a general issue, relating not only to Relativity or the Quanta, the presence of infinitesimal and infinitely large scalars in reduced power algebras may correspond to a new possibility of having no less than *two* radically different kind of measurements when it comes to their *relative precision*.

Namely, one has an *increased precision* in measurement, when measurement is done in terms of usual finite scalars, and one obtains as result some infinitesimal scalar in such algebras.

Alternatively, the presence of infinitely large scalars in such algebras may simply indicate that they were obtained in terms of finite scalars, and thus are but the result of a measurement with *decreased precision*.

In this regard, we can therefore have the following *relative* situations

- infinitesimal scalars are the result of increased precision measurements done in terms of finite or infinite scalars,
- finite scalars are the result of increased precision measurements done in terms of infinite scalars,
- finite or infinitely large scalars are the result of decreased precision measurements done in terms of infinitesimal scalars,
- infinitely large scalars are the result of decreased precision measurements done in terms of infinitesimal or finite scalars.

and surprisingly, one can also have the following *relative* situations

- infinitesimal scalars are the result of increased precision measurements done in terms of some less infinitesimal scalars,
- infinitesimal scalars are the result of decreased precision measurements done in terms of some more infinitesimal scalars,
- infinitely large scalars are the result of increased precision measurements done in terms of some more infinitely large scalars,

- infinitely large scalars are the result of decreased precision measurements done in terms of some less infinitely large scalars.

Indeed, one of the basic features of reduced power algebras is precisely their complicated and rich *self-similar* structure which distinguishes not only between infinitesimal, finite and infinitely large scalars, but also within the infinitely small scalars themselves, and similarly, within the infinitely large scalars. Specifically, infinitesimal scalars can be infinitely smaller, or on the contrary, infinitely larger than other infinitesimals. And similarly, infinitely large scalars can be infinitely smaller, or on the contrary, infinitely larger than other infinitely large scalars.

Here, however, we can note that such a possible interpretation of increased, or decreased precision which is *relative*, is in fact not new. Indeed, in terms of usual scalars, be they real or complex, there is a marked dichotomy between finite scalars, and on the other hand, the so called "infinities" which may on occasion arise from operations with finite scalars. And such simple "formulas" like $\infty + 1 = \infty$, are in fact expressing that fact. Namely, on one hand, from the point of view of "infinity", the finite number 1 has such an increased precision as to be irrelevant with respect to addition, while on the other hand, from the point of view of the finite number 1, the "infinity" has such a decreased precision as to alter completely the result when involved in addition.

3.4. The Issue of Universal Constants

Given the above possibilities in interpretation leading to relative precision measurement - be it as such an increased or a decreased one - one can reconsider the status of certain universal physical constants, such as for instance, the Planck constant h and the constant c giving the velocity of light in vacuum.

Indeed, when considered from our everyday macroscopic experience, h is supposed to be unusually small, while on the contrary, c is very large. Consequently, one may see h as a sort of "infinitesimal", while c then looks like "infinitely large".

The fact is that, within reduced power algebras, such an alternative view of h and c is possible. Therefore, one may find it appropriate to explore the possible physical meaning, or otherwise, that may possibly be associated with such an interpretation.

Appendix : Zero Divisors, Units and other Properties in Reduced Power Algebras

A.1. Construction of Reduced Power Algebras

The general construction of *reduced power algebras* goes as follows, [2-11]. Let Λ be any *infinite* set. Let \mathcal{F} be any filter on Λ , such that

$$(A.1.1) \quad \mathcal{F}_{re}(\Lambda) \subseteq \mathcal{F}$$

where

$$(A.1.2) \quad \mathcal{F}_{re}(\Lambda) = \{ I \subseteq \Lambda \mid \Lambda \setminus I \text{ is finite} \}$$

is called the Frechét filter on Λ .

We define on \mathbb{R}^Λ the corresponding equivalence relation $\approx_{\mathcal{F}}$ by

$$(A.1.3) \quad x \approx_{\mathcal{F}} y \iff \{ \lambda \in \Lambda \mid x(\lambda) = y(\lambda) \} \in \mathcal{F}$$

where $x, y \in \mathbb{R}^\Lambda$.

Then, through the usual quotient construction, we obtain the *reduced power algebra*

$$(A.1.4) \quad \mathbb{A}_{\mathcal{F}} = \mathbb{R}^\Lambda / \approx_{\mathcal{F}}$$

which has the following two properties.

The mapping

$$(A.1.5) \quad \mathbb{R} \ni r \longmapsto (u_r)_{\mathcal{F}} \in \mathbb{A}_{\mathcal{F}}$$

is an *embedding of algebras* in which \mathbb{R} is a strict subset of $\mathbb{A}_{\mathcal{F}}$, where $u_r \in \mathbb{R}^{\Lambda}$ is defined by $u_r(\lambda) = r$, for $\lambda \in \Lambda$, while $(u_r)_{\mathcal{F}}$ is the coset of u_r with respect to the equivalence relation $\approx_{\mathcal{F}}$.

Further, on $\mathbb{A}_{\mathcal{F}}$ we have the *partial order* which is compatible with the algebra structure, namely

$$(A.1.6) \quad (x)_{\mathcal{F}} \leq (y)_{\mathcal{F}} \iff \{ \lambda \in \Lambda \mid x(\lambda) \leq y(\lambda) \} \in \mathcal{F}$$

where $x, y \in \mathbb{R}^{\Lambda}$.

As is well known

$$(A.1.7) \quad \mathbb{A}_{\mathcal{F}} \text{ is a field} \iff \mathcal{F} \text{ is an ultrafilter on } \Lambda$$

consequently

$$(A.1.8) \quad \mathbb{A}_{\mathcal{F}} \text{ has zero divisors} \iff \mathcal{F} \text{ is not an ultrafilter on } \Lambda$$

It will be useful to consider the *non-negative* elements in $\mathbb{A}_{\mathcal{F}}$, given by

$$(A.1.9) \quad \mathbb{A}_{\mathcal{F}}^+ = \{ (x)_{\mathcal{F}} \mid x \in \mathbb{R}, \quad \{ \lambda \in \Lambda \mid x(\lambda) \geq 0 \} \in \mathcal{F} \}$$

A.2. Zero Divisors and Units in $\mathbb{A}_{\mathcal{F}}$

Let \mathcal{F} be a filter on Λ which satisfies (A.1.1) and is not an ultrafilter on Λ . Given any $x \in \mathbb{R}^{\Lambda}$, we denote

$$(A.2.1) \quad Z(x) = \{ \lambda \in \Lambda \mid x(\lambda) = 0 \} \subseteq \Lambda$$

and obviously, we have the following four alternatives

$$(A.2.2.1) \quad Z(x) \in \mathcal{F}$$

$$(A.2.2.2) \quad Z(x) \notin \mathcal{F}$$

$$(A.2.2.3) \quad \Lambda \setminus Z(x) \in \mathcal{F}$$

$$(A.2.2.4) \quad \Lambda \setminus Z(x) \notin \mathcal{F}$$

Since \mathcal{F} is not an ultrafilter, alternatives (A.2.2.1) and (A.2.2.3) are not incompatible. Therefore, the same applies to alternatives (A.2.2.2) and (A.2.2.4). It follows that we have the mutually exclusive four alternatives

$$(A.2.3.1) \quad Z(x) \in \mathcal{F} \text{ and } \Lambda \setminus Z(x) \in \mathcal{F}$$

$$(A.2.3.2) \quad Z(x) \in \mathcal{F} \text{ and } \Lambda \setminus Z(x) \notin \mathcal{F}$$

$$(A.2.3.3) \quad Z(x) \notin \mathcal{F} \text{ and } \Lambda \setminus Z(x) \in \mathcal{F}$$

$$(A.2.3.4) \quad Z(x) \notin \mathcal{F} \text{ and } \Lambda \setminus Z(x) \notin \mathcal{F}$$

Now in view of (A.1.3), we have

$$(A.2.4) \quad Z(x) \in \mathcal{F} \iff (x)_{\mathcal{F}} = 0 \in \mathbb{A}_{\mathcal{F}}$$

thus alternatives (A.2.3.1) and (A.2.3.2) are clarified in their consequence.

Let us now consider (A.2.3.3) and define $y \in \mathbb{R}^{\Lambda}$ by

$$(A.2.5) \quad y(\lambda) = \begin{cases} 1/x(\lambda) & \text{if } \lambda \in \Lambda \setminus Z(x) \\ \text{arbitrary} & \text{otherwise} \end{cases}$$

then (A.1.3), (A.2.4) give

$$(A.2.6) \quad (x)_{\mathcal{F}}, (y)_{\mathcal{F}} \neq 0 \in \mathbb{A}_{\mathcal{F}}, \quad (x)_{\mathcal{F}}(y)_{\mathcal{F}} = 1 \in \mathbb{A}_{\mathcal{F}}$$

thus $(x)_{\mathcal{F}}$ is an *invertible element*, or a *unit* in $\mathbb{A}_{\mathcal{F}}$, and $((x)_{\mathcal{F}})^{-1} = (y)_{\mathcal{F}}$.

In the case of (A.2.3.4), let us define $y \in \mathbb{R}^{\Lambda}$ by

$$(A.2.7) \quad y(\lambda) = \begin{cases} 0 & \text{if } \lambda \in \Lambda \setminus Z(x) \\ 1 & \text{if } \lambda \in Z(x) \end{cases}$$

then (A.1.3), (A.2.4) give

$$(A.2.8) \quad (x)_{\mathcal{F}}, (y)_{\mathcal{F}} \neq 0 \in \mathbb{A}_{\mathcal{F}}, \quad (x)_{\mathcal{F}}(y)_{\mathcal{F}} = 0 \in \mathbb{A}_{\mathcal{F}}$$

thus $(x)_{\mathcal{F}}$ is a *zero divisor* in $\mathbb{A}_{\mathcal{F}}$.

It follows that the set of *units*, or *invertible elements* in $\mathbb{A}_{\mathcal{F}}$ is given by

$$(A.2.9) \quad \mathbb{A}_{\mathcal{F}}^u = \{ (x)_{\mathcal{F}} \mid x \in \mathbb{R}^{\Lambda}, Z(x) \notin \mathcal{F}, \Lambda \setminus Z(x) \in \mathcal{F} \}$$

while the set of *zero divisors* in $\mathbb{A}_{\mathcal{F}}$ is given by

$$(A.2.10) \quad \mathbb{A}_{\mathcal{F}}^{zd} = \{ (x)_{\mathcal{F}} \mid x \in \mathbb{R}^{\Lambda}, Z(x) \notin \mathcal{F}, \Lambda \setminus Z(x) \notin \mathcal{F} \}$$

and clearly, we have the following partition in three disjoint subsets

$$(A.2.11) \quad \mathbb{A}_{\mathcal{F}} = \{0\} \cup \mathbb{A}_{\mathcal{F}}^{zd} \cup \mathbb{A}_{\mathcal{F}}^u$$

A.3. Infinitesimals and Infinitely Large Scalars

The reduced power algebras $\mathbb{A}_{\mathcal{F}}$ contain strictly as a subfield the field \mathbb{R} of usual real numbers. In addition, the reduced power algebras $\mathbb{A}_{\mathcal{F}}$ contain vast amounts of *infinitesimal*, as well as *infinitely large* scalars.

In case in (A.1.3), and in the sequel, we replace \mathbb{R} with \mathbb{C} , and thus \mathbb{C}^{Λ} takes the place of \mathbb{R}^{Λ} , then we obtain reduced power algebras which contain strictly the field \mathbb{C} of usual complex numbers. And again, the reduced power algebras will contain vast amounts of *infinitesimal*, as well as *infinitely large* scalars.

A.4. Reduced Power Fields

The following properties are equivalent :

(A.4.1) \mathcal{F} is an *ultrafilter* on Λ

(A.4.2) $\mathbb{A}_{\mathcal{F}}^{zd} = \phi$, $\mathbb{A}_{\mathcal{F}} = \{0\} \cup \mathbb{A}_{\mathcal{F}}^u$ is a field

(A.4.3) For every $x \in \mathbb{R}^\Lambda$, the four alternatives (A.2.3.1) - (A.2.3.4) reduce to the following two, namely, (A.2.3.2), (A.2.3.3), that is :

$$Z(x) \in \mathcal{F} \text{ and } \Lambda \setminus Z(x) \notin \mathcal{F}$$

$$Z(x) \notin \mathcal{F} \text{ and } \Lambda \setminus Z(x) \in \mathcal{F}$$

(A.4.4) The partial order $\leq_{\mathcal{F}}$ in (A.1.6) is a linear, or total order on the reduced power field $\mathbb{A}_{\mathcal{F}}$

A.5. Exponential Functions

In view of (A.1.9), one can obviously define the exponentiation

$$(A.5.1) \quad \mathbb{A}_{\mathcal{F}}^+ \times \mathbb{A}_{\mathcal{F}}^+ \ni ((x)_{\mathcal{F}}, (y)_{\mathcal{F}}) \longmapsto (z)_{\mathcal{F}} = ((x)_{\mathcal{F}})^{((y)_{\mathcal{F}})} \in \mathbb{A}_{\mathcal{F}}^+$$

by

$$(A.5.2) \quad z(\lambda) = (x(\lambda))^{(y(\lambda))}, \quad \lambda \in I$$

where $I \in \mathcal{F}$ is such that

$$(A.5.3) \quad x(\lambda), y(\lambda) \geq 0, \quad \lambda \in I$$

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